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P1_3 Relativistic Optics

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Abstract

The famous 1970s motion picture *Star Wars* depicts the stars as stretched beams of light passing the view of the *Millennium Falcon* as it nears the speed of light. Investigations are undertaken into what an interstellar traveller would observe during their journey due to relativistic optics. It is concluded that the traveller would observe a central disc of aberrated radiation, whereby the frequencies are all blue-shifted; the Cosmic Microwave Background Radiation peaks in intensity at λ~530nm, and visible light from stars peaks in the X-ray range (0.20 – 0.35nm).

Introduction

A visit to other stars in the Galaxy in a human lifetime would involve very high relativistic γ-factors. The famous 1970s motion picture *Star Wars* depicts the stars as stretched beams of light passing the view of the *Millennium Falcon* as it reaches velocities close to the speed of light [1] (a speed of v=0.9999995c is assumed throughout this paper). This article assumes an advanced civilisation could achieve these velocities and outlines what a voyager would observe as they travel on their interstellar journeys, paying particular attention to stellar radiation and the Cosmic Microwave Background Radiation (CMBR).

Relativistic Doppler Shift and Aberration

Waves from an approaching light-source have higher frequencies than waves from a stationary source [2]. This effect, known as the Doppler Effect, becomes significantly large when the relative motion between the source and the observer approaches relativistic velocities (the effect becomes a relativistic one). We therefore consider how the frequency and angular shape of the radiation observed by an interstellar traveller would change due to the velocity of travel. It is useful at this point to define frames of reference: consider the observer to be the interstellar traveller, in a frame S', and the isotropic source to be any form of radiation emitted in the field of view of the observer, in a frame S, with the isotropic source placed at the origin (see *Figure 1*). The relative velocity between the two frames is given by v, the velocity of the traveller, whereby frame S' moves towards frame S. Since v=0.9999995c, any dispersive motions of radiation sources, for example the orbital velocities of stars, are neglected. The transformation of photon energies between frames is calculated using the four-vector of the momentum,

$$\underline{\boldsymbol{P}} = \hbar \begin{pmatrix} \omega/c \\ k\cos\alpha \\ k\sin\alpha \\ 0 \end{pmatrix}, \qquad (1)$$

where ω is the angular frequency of the photon, k is the photon wave vector, α is the angle the photon makes with the x-axis, and $\hbar$ is Planck's constant.

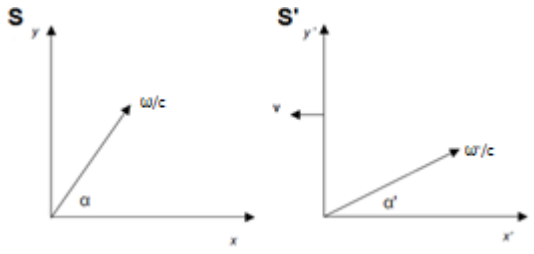

Figure 1- The transformation between the two frames of reference *S* and *S'* with the source at the origin. The two frames of reference differ by a velocity *v* and α is the angle the photon makes with the x-axis. Diagram edited from [3].

The photon emission process is considered to be in the x-y plane since the effects are symmetrical about this plane for the observer. All components of the four-vector have units of momentum, with the first term being the intrinsic momentum of the photon and the three terms following being the components of the momentum in the x, y, and z directions, respectively. The transformation of *Equation (1),* from frame S to S' (moving at velocity v toward S), is found by multiplying by the Lorentz matrix as shown in *Equation (2)* [2],

$$\underline{\boldsymbol{P'}} = \hbar \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} {}^{\omega}/_{c} \\ k\cos\alpha \\ k\sin\alpha \\ 0 \end{pmatrix}, \quad (2)$$

where γ is the Lorentz factor ($\gamma = \frac{1}{\sqrt{1-\beta^2}}$), and $\beta = {}^{v}/_{c}$. *Equation (2)*, along with the identity ${}^{\omega}/_{k} = c$, yields the relations for Doppler shift and aberration, respectively;

$$\frac{\omega'}{\omega} = \gamma(1 + \beta\cos\alpha), \quad (3)$$

and,

$$\tan\alpha' = \frac{1}{\gamma}\left[\frac{\sin\alpha}{\cos\alpha+\beta}\right]. \quad (4)$$

The angular position and frequency of the light from a radiative source can now be calculated in the frame of the observer travelling toward the source at a velocity v. *Figure 2* shows a graphical representation of the angular shift of impinging radiation from an isotropic source, plotted using *Equation (4)*. It can be seen that the light is aberrated into the forward direction into a cone of light, with the intensity decreasing radially outward.

The blackbody temperature of the CMBR is measured to be 2.73K [4], giving a maximum wavelength of emitted radiation of λ=1.06mm (given by Wien's displacement law) [5]. The wavelength of a photon (using *Equation (3)* and $c=\omega\lambda/2\pi$) travelling along the x-axis toward an interstellar traveller is λ'=530nm, which falls in the visible light range of the electromagnetic spectrum. Wien's displacement law also gives a CMBR blackbody temperature of T'=5468K. By the same procedure the light emitted from stars in the visible range (400nm– 700nm) is blue-shifted into the X-ray range, λ'=0.20–0.35 nm.

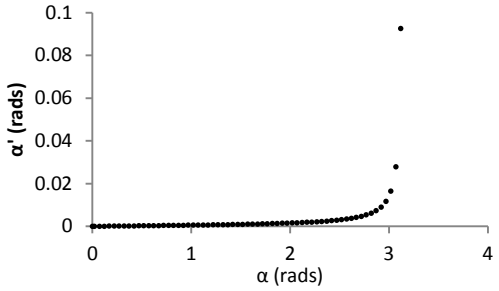

Figure 2- The relation between the emission angle of radiation α from an isotropic point source in frame S and the angle α' as seen by an observer in frame S'.

Conclusion

It is concluded that the radiation in the field of view of an interstellar traveller will appear as a single cone of light, with the highest intensity observed at the centre of this cone, decreasing radially outwards. All the radiation observed is blue-shifted into a higher frequency range causing the CMBR to become visible (at 400nm–700nm) and the radiation from stars shifted into the X-ray range (0.20–0.35 nm). Stars will appear to exit the cone as the observer travels at relativistic speeds but the CMBR will always be present as a disc of light in the direction of travel. As such, this topic raises further questions into how relativistic speeds will affect interstellar travel (i.e. drag effects from such high intensity radiation and the need for protection against this highly energetic radiation).

References

[1]http://www.blueharvest.net/scoops/anh-script.shtml [Accessed on 30/10/12]

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[3]http://www.triangulum.nl/Werkgroepen/documentatie%20werkgroepen/Astrofysica/aberration%20essay.pdf [Accessed on 30/10/12]

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[5]Tipler P and Mosca G, *Physics for Scientists and Engineers 6ed*, W.H. Freeman & Company, pg684 (2008)